PHYS221
Uncertainty notes-Torque Experiment

Let's assume you have measured the following measurements (see fig. 9-2 of procedure for torque notation):

Left hand side of fulcrum- mass = (masses + hanger)= (0.200 + 0.01660) kg = **0.21660 kg**
distance from fulcrum = 35.6 cm = **0.356 m**

Right hand side of fulcrum - (masses + hanger) = (0.300 + 0.01650) kg = **0.31665 kg**
distance from fulcrum = 24.4 cm = **0.244 m**

These measurements yield the following torques, respectively;

$$\tau_{cc} = \left(0.21666kg \times 9.8000\ \frac{m}{s^2} \times 0.356m\right) = 0.75588\ N\text{*m}, \quad \text{and}$$

$$\tau_c = -\left(0.31665kg \times 9.8000\ \frac{m}{s^2} \times 0.244m\right) = -0.75717\ N\text{*m}$$

We must now determine the uncertainty of the torque using the following uncertainties:

We are using a meter stick with a readability of 1 mm and an uncertainty- $\delta r = 0.5mm = 0.05cm$
However, since there are two clamps there is an uncertainty associated with both and thus we will use an uncertainty of $\delta r = \sqrt{0.05^2 + 0.05^2}\,cm = \sqrt{2} \times 0.05cm = 0.07cm$

Uncertainty of r use $0.07cm$	For the **mass uncertainty** we will use 1% of total mass

(Please note that this value for mass uncertainty is an approximate method only and there are several different ways to achieve a more precise value, e.g., using a balance to measure individual masses or by using a statistically determined mass uncertainty)

Since torque is defined as force times distance (i.e., $\tau = F \times d$) you must use uncertainty of multiplied quantities (see appendix –pages A-6 through A-10). Thus we see

$$\frac{\delta\tau}{\tau} = \sqrt{\left(\frac{\delta F}{F}\right)^2 + \left(\frac{\delta r}{r}\right)^2} = \sqrt{\left(\frac{\delta(mg)}{mg}\right)^2 + \left(\frac{\delta r}{r}\right)^2}.$$

Since force (weight) is mg and we will assume g is a constant (with an ignorable uncertainty), we see

$$\frac{\delta\tau}{\tau} = \sqrt{\left(\frac{\delta(mg)}{mg}\right)^2 + \left(\frac{\delta r}{r}\right)^2} = \sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta r}{r}\right)^2}. \quad \Rightarrow$$

$$\frac{\delta\tau}{\tau} = \sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta r}{r}\right)^2} \qquad \text{Eq-1}$$

Please note the equation 1above is simply another way of saying that the fractional uncertainty of quantities that are multiplied together is obtained by **first converting uncertainties to a fraction** (or percent) and **then adding those uncertainties** (in this case we are adding in quadrature).

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Examining the measured values above, we see for counterclockwise values;

$$\frac{\delta\tau_{cc}}{\tau_{cc}} = \sqrt{\left(\frac{\delta m_{cc}}{m_{cc}}\right)^2 + \left(\frac{\delta r_{cc}}{r_{cc}}\right)^2} \quad \Rightarrow \quad \frac{\delta\tau_{cc}}{0.75588 N\text{-}m} = \sqrt{\left(\frac{(2.17g)}{216.66g}\right)^2 + \left(\frac{(0.07cm)}{35.6cm}\right)^2}$$

Please note that 1% of 216.66g is 2.17g

$$\delta\tau_{cc} = 0.75588\ N * m \times \sqrt{(0.01)^2 + (0.0019663)^2} = 0.75588\ N * m * 0.0101915 = 0.0077\ N * m$$

Rounding the uncertainty to one significant figure we see

$\delta\tau_{cc} = 0.008\ N*m.$

We can thus write counter clockwise torque sum and its uncertainty (i.e., $\tau_{cc} \pm \delta\tau_{cc}$) as

$$(0.75588\ \pm 0.008)N * m = (0.756 \pm 0.008)N * m$$

Repeating the process above for the clockwise torque we see

$$\frac{\delta\tau_c}{0.75717\ N * m} = \sqrt{\left(\frac{3.17g}{316.65g}\right)^2 + \left(\frac{0.07cm}{24.4cm}\right)^2}$$

which yields

$$\delta\tau_c = 0.007877\ N * m = 0.008\ N * m$$

We can thus write clockwise torque sum and its uncertainty (i.e., $\tau_c \pm \delta\tau_c$) as

$$(-0.757178\ \pm 0.008)N * m = (-0.757\ \pm 0.008)Nm$$

The sum of the torques (including uncertainties) is given by

$$\Sigma\tau = (0.756 \pm 0.008)N * m + (-0.757\ \pm 0.008)N * m$$

Please note that the uncertainties should be added in quadrature to yield final answer

$$\Sigma\tau = \left(-0.001 \pm \sqrt{0.008^2 + 0.008^2}\right)N * m = -0.001 \pm 0.011\ N * m$$

Writing the uncertainty with one significant figure yields a final answer of

$$\Sigma\tau = (-0.00 \pm 0.01)N * m$$